FOCUSED ISSUE FEATURE

Getting Its Measure

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Accurate measurement of phase noise is one of the most difficult measurements in all of electrical engineering. The biggest challenge is the huge dynamic range required in most phase noise measurements. There are several methods to measure phase noise, and the right one must be chosen to make the necessary measurements. To make a proper selection from the various methods, it is necessary to know and appreciate the weaknesses and strengths of each of the different techniques because none of these methods is perfect for every situation [1]–[44]. This article focuses on key phase noise measurement techniques for oscillators and reviews their advantages and disadvantages.

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In general, measuring phase noise is more difficult than measuring amplitude or frequency related properties. Different signal sources, whether it is an oscillator alone or within a synthesizer, have widely varying phase noise performances. Higher noise sources do not work well with phase noise measurement equipment optimized to measure very low noise levels. An ability to measure the phase noise performance of ultra-low phase noise oscillators drives the specifications of the best performing phase noise analyzers.

Phase noise is usually expressed in units of dBc/Hz at some specific offset frequency f_s from the carrier, the value of the noise level relative to the carrier level calculated in 1 Hz bandwidth. Most often only single sideband (SSB) noise is considered. Some measurement set-ups measure both noise sidebands and a conversion factor is required to report SSB noise.

The pioneer in phase noise measurement unquestionably was Hewlett Packard [1].

Once adequate for advanced designs, a noise floor dictated by SSB thermal noise (Johnson noise at kT) of −174 dBm/Hz for zero dBm output power is not enough anymore for some special requirements and also marketing of these reference frequency sources. The noise correlation technique allows us to look below kT level (< −174 dBm/Hz). But the usefulness of the noise contribution below kT is debatable in the perspective of overall system performance. To achieve a very low measurement noise floor, many modern phase noise measurement instruments use the correlation principle, with all its pros and cons as it is described in subsequent sections [41]–[43].

Phase Noise Measurement Techniques

The usual goal for measuring phase noise in an R&D environment is to achieve the lowest measurement noise floor possible. As we shall see, this is not necessarily the best choice, depending on the signal source being measured. In a production environment, the objective is fast throughput for product phase noise performance testing. Again, this is best achieved by using a method that is appropriate for the source being measured.

There are some very capable general purpose phase noise measurement instruments available on the market, including the Agilent E5052B, Rhode & Schwarz FSU 26, Holzworth, Noise XT, and Anapico APPH6000-IS. With the growing demand for improved

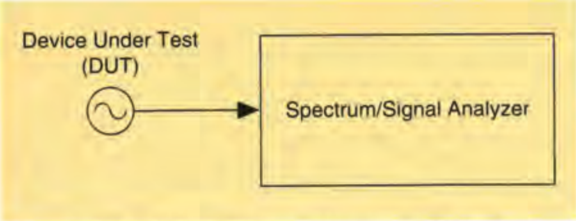

Figure 1. *The direct spectrum measurement technique.*

dynamic range and lower noise floor, equipment companies are introducing general purpose phase noise analysis software-driven tools for extracting far out (offset frequency > 1 MHz) noise below the kT floor even though claims of −195 dBc/Hz or lower lack the practical utility.

Modern phase noise test equipment addresses these issues, but one must understand the limitations of measurement techniques so that a suitable method can be chosen. The direct spectrum method, phase lock loop (PLL) method, delay-line discriminator method, and cross-correlation method are frequently used to measure the oscillator phase noise. The first one is the simplest and has the biggest limitation. The last one requires the most complex measurement system but it is a versatile one that can measure oscillator phase noise performance better than that of its reference oscillator.

Here we present the following primary phase noise measurement techniques, listed in the order of increasing precision:

- direct spectrum technique
- frequency discriminator method
  - heterodyne (digital) discriminator method
- phase detector (PD) techniques
  - (reference source/PLL method)
- residual method
- two-channel cross-correlation technique.

Direct Spectrum Technique

This is the simplest technique for making phase noise measurements. Using this technique, measurements are valid as long as the analyzer's phase noise is significantly lower than that of the measured device. Figure 1 shows the basic block diagram of a direct spectrum measurement technique. As shown in Figure 1, the signal from the device under test (DUT) is input into a spectrum/signal analyzer tuned to the DUT frequency, directly measuring the power spectral density of the oscillator in terms of $\pounds(f_m)$. Because the spectral density is measured with the carrier present, this method is limited by the spectrum/signal analyzer's dynamic range. Though this method may not be useful for measuring very close-in phase noise to a drifting carrier, it is convenient for qualitative quick evaluation of sources with relatively high noise. For practical application, the measurement is valid if the following conditions are met:

- The spectrum/signal analyzer's internal SSB phase noise at the offset of interest must be lower than the noise of the DUT. It is therefore essential to know the internal phase noise of the analyzer you are using.
- Because the spectrum/signal analyzer measures total noise power without differentiating amplitude noise from phase noise, the amplitude noise of the DUT must be significantly below its phase noise (typically 10 dB will suffice). This can be assured

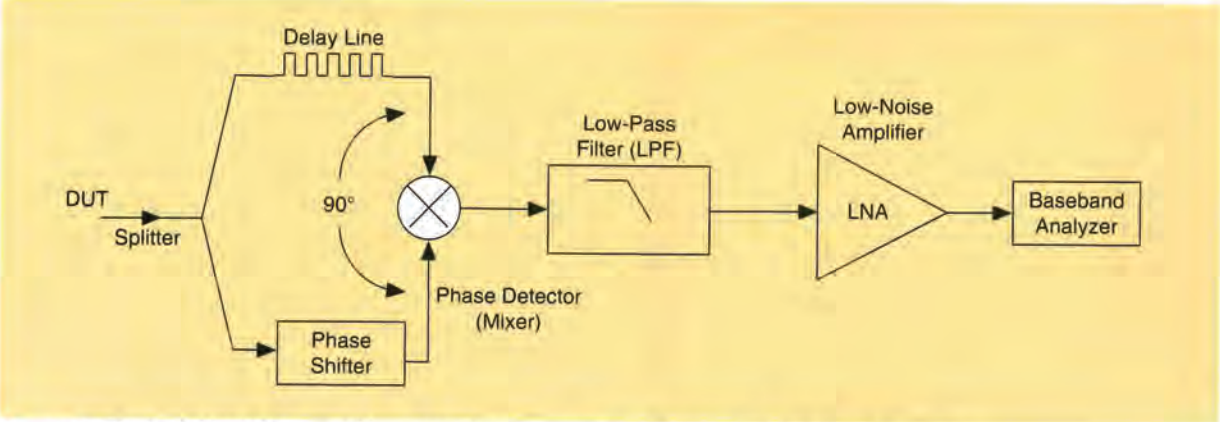

Figure 2. *The basic block diagram of frequency discriminator method (courtesy: Agilent Company).*

by first passing the DUT signal through a limiter. The presence of amplitude noise is suggested if the sidebands of the signal are not symmetrical.

It is very important to adjust the noise measurement from the spectrum analyzer. All spectrum analyzers pass signals through a logarithmic amplifier (logamp) before detection and averaging. This distorts the noise waveform, essentially clipping it somewhat from the logarithmic transfer function. A 2.5 dB error on the low side results from this average-of-log process. See Agilent application note AN1303, "Spectrum and Signal Analyzer Measurements and Noise" [1] for more details.

Advantages

- Simple, frequency based measurement
- Fast measurement, for relatively noisy sources
- Relatively low cost.

Suitable for measurements of oscillators that drift slightly (less than the resolution filter bandwidth) during measurement.

Drawbacks

- Not suitable for measuring oscillators with ultra low phase noise performance, because the noise floor of the instrument is comparatively high.
- Not suitable for measuring the phase noise within 1 kHz carrier frequency, mostly because spectrum analyzers have their own noise properties that can degrade the measurement results.
- Limited measurement dynamic range.

One of the major drawbacks of the direct spectrum technique is its dynamic range limitation due to the presence of the carrier power. All of the following measurement techniques eliminate this limitation by separating the sideband noise from the carrier power, using a variety of techniques.

Frequency Discriminator Method

In the frequency discriminator method, the frequency fluctuations of the source are translated to low frequency voltage fluctuations that can then be measured by a baseband analyzer. There are several common implementations of frequency discriminators including cavity resonators, RF bridges, and a delay line.

Delay-Line Frequency Discriminator

The delay-line measurement system is often chosen for the flexibility in measuring a free-running oscillator between 1 GHz and 10 GHz. The delay-line technique has sufficient sensitivity to measure most microwave oscillators with loaded Q-factors of several hundred and does not require a second reference oscillator.

The expression of delay in a transmission line can be calculated as [31]–[32]

$$t_{\text{delay}} = \sqrt{\varepsilon_r}\frac{l_{\text{cable}}}{c}, \tag{1}$$

where ε_r is the relative dielectric constant in a coaxial cable.

The primary advantage of this method is that it can be used to measure noisy sources, but on the other hand, it does not work with low noise sources because the noise performance of this method is the limiting factor [31]. Delay-line discriminators are limited by the loss of the delay-line due to the power requirements for the mixer. Using lower power than required will lead to degraded performance of the system. The noise floor depends on the length of the cable (delay); the longer the delay, the lower the noise floor, but it will also mean higher losses and lower offset frequency. The highest usable offset frequency depends mostly on the length of the delay. There is a response null at $f = 1/t_{\text{delay}}$ offset frequency, and the recommendation is to use offset frequencies up to $f = 1/(4t_{\text{delay}})$. With a 500 ns delay, the usable offset frequency range is from 0 to 500 kHz.

As shown in Figure 2, the signal power from the DUT is split into two channels. The signal in one path is delayed relative to the signal in the other path. The delay line converts the frequency fluctuation to phase fluctuation. The mixer requires phase quadrature at its two inputs at the carrier frequency, which is achieved by either adjusting the delay line (not likely) or using

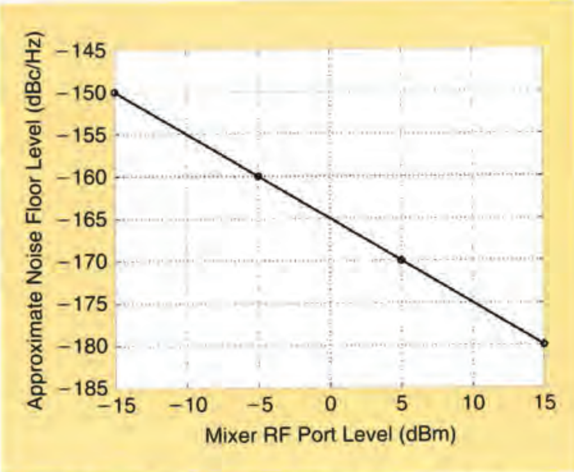

Figure 3. *The ideal PD sensitivity in terms of RF power (assuming LO power is great than RF) and PD constant $K\phi$. The noise floor sensitivity is 1:1 to mixer power input [31].*

a small phase shifter in the through-path. As shown in Figure 2, the mixer (acting as PD) converts the phase difference between the delayed and undelayed paths into a dc voltage related by the phase discriminator constant K_ϕ. The output of this frequency discriminator is then read on the baseband spectrum analyzer as frequency noise. This frequency noise is converted to phase noise using the well-known relationship between FM and PM, and reported as phase noise measurement.

The frequency fluctuations of the oscillator in terms of offset frequency f_m are related to the PD constant K_ϕ and the delay τ_d by [31]:

$$\Delta V(f_m) = [K_\phi 2\pi\tau_d]\Delta f(f_m) = K_d \Delta f(f_m). \tag{2}$$

Since frequency is the time rate change of phase we have:

$$S_\phi(f_m) = \frac{S_{\Delta f}(f_m)}{f_m^2} = \frac{\Delta f^2(f_m)}{f_m^2}. \tag{3}$$

The voltage output is measured as a double sideband voltage spectral density $S_v(f_m)$.

From (2) and (3), phase noise $S_\phi(f_m)$ is related to the measured $S_v(f_m)$ by:

$$S_\phi(f_m) = \frac{\Delta V^2(f_m)}{K_d^2 f_m^2} = \frac{S_v(f_m)}{K_d^2 f_m^2}. \tag{4}$$

The single sideband phase noise is given by

$$\mathcal{L}_\phi(f_m) = \frac{S_v(f_m)}{2K_d^2 f_m^2} \tag{5a}$$

$$\mathcal{L}_\phi(f_m)[\text{dBc/Hz}] = S_v(f_m) - 3 - 20\log(K_d) - 20\log(f_m). \tag{5b}$$

With a single calibration of the mixer as a PD, K_ϕ and known delay τ_d, the phase noise of an oscillator can be measured using FFT (baseband) analyzer. The phase discriminator constant K_ϕ is in V/rad and is determined by measuring the dc output voltage change of a mixer while in quadrature (nominally 0 V dc) for a known phase change in one branch of discriminator. The value of K_d is dependent upon the RF input power of the mixer that in turn is directly proportional to the noise floor shown in Figure 3 [31]. Using Z-parameters the sensitivity of the delay-line discriminator can be determined first by introducing the Q-factor defined with respect to the phase of the open-loop transfer function $\phi(\omega)$ at the resonance of parallel RLC circuit [31]–[33]:

$$\phi(j\omega) = \tan^{-1}\frac{\text{Imag}(Z(j\omega))}{\text{Real}(Z(j\omega))} \tag{6}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\omega\delta_\phi}{2\delta\omega}. \tag{7}$$

A typical coaxial delay-line exhibits a linear phase relation with frequency across the usable bandwidth of the transmission line.

The linear phase relationship in a coaxial line to the derivative of the phase change in a resonator results in an effective Q, Q_E for a transmission line with time delay τ_d:

$$Q_E = \pi f_0 \tau_d. \tag{8}$$

From (8), the effective Q-factor increases linearly with both delay-line length and frequency of operation. Using Q_E as the Q-factor in the Leeson's equation and using an approximate mixer noise floor of −175 dBc. The Flicker corner is set at 10 kHz, typical for a silicon diode mixer. The measurement phase noise floor is calculated by:

$$\mathcal{L}_\phi(f_m) = 10\log\left[\left(1 + \frac{1}{(2\pi\tau_d f_m)^2}\right)\left(1 + \frac{f_c}{f_m}\right)\right] + N_{\text{mixer floor}}. \tag{9}$$

A plot of (9) is shown in Figure 3.

Advantages

- Better sensitivity than direct spectrum methods
- Good for free running sources such as LC oscillators or cavity oscillators
- Appropriate when the DUT is a relatively noisy source with high-level, low rate phase noise or high close-in spurious sideband.

Drawbacks

- Significantly less sensitivity than PD methods.
- A longer delay line will improve the sensitivity but the insertion loss of the delay line may exceed the source power available and cancel any further improvement.

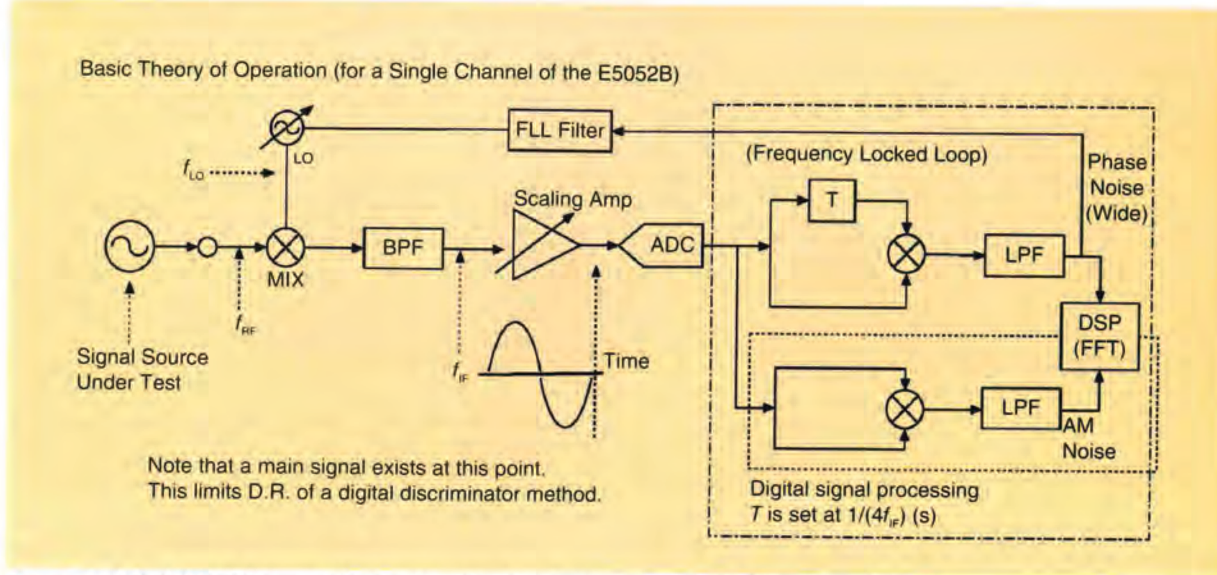

Figure 4. *A basic block diagram of heterodyne (digital) discriminator method (courtesy: Agilent).*

- Also, longer delay lines limit the maximum offset frequency that can be measured. This method is best used for free running sources such as LC oscillators or cavity oscillators. Although the frequency discriminator method degrades the measurement sensitivity (at close-in offset frequency, in particular).

Heterodyne (Digital) Discriminator Method

Shown in Figure 4, the heterodyne (digital) discriminator method is a modified version of the analog delay-line discriminator method and can measure the relatively large phase noise of unstable signal sources and oscillators. Unlike the analog discriminator method, here the input signal is downconverted to a fixed intermediate frequency f_{if} using a separate local oscillator. The local oscillator is frequency locked to the input signal. Working at a fixed frequency, the frequency discriminator does not need reconnection of various analog delay lines at any frequency. This method features wider phase noise measurement ranges than the PLL method and.

This option is available in latest version of phase noise measurement equipment (Agilent E5052B).

Advantages

- Offers easy and accurate AM noise measurements (by setting the delay time to zero) with the same setup and RF port connection as the phase noise measurement
- Frequency demodulation can be implemented digitally.

Drawbacks

- Dynamic range of phase noise measurement is further limited by the additional scaling amplifier and ADCs.

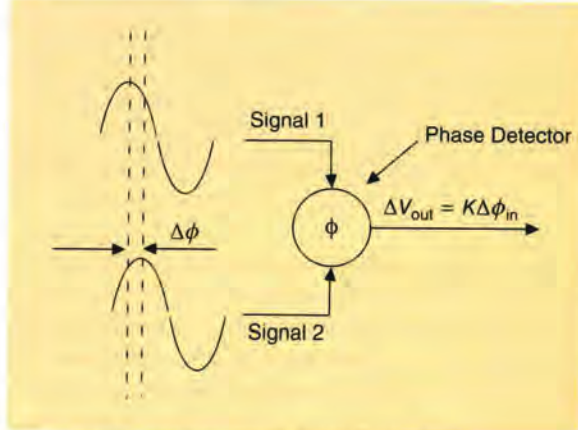

Figure 5. *A basic concept of PD techniques.*

Phase Detector Technique

Figure 5 shows the basic concept for the PD technique. The PD method measures voltage fluctuations directly proportional to the combined phase fluctuations of the two input sources. To separate phase noise from amplitude noise, a PD is required. The PD converts the phase difference of the two input signals into a voltage at the output of the detector. When the phase difference between the two input signals is set to 90° (e.g. at quadrature), the nominal output voltage is zero volts and sensitivity to AM noise is minimized. Any phase fluctuation from quadrature results in voltage fluctuation at the output. This method has a very low noise floor and therefore has a very good measurement dynamic range.

Reference Source/PLL Method

Figure 6 shows the basic block diagram of the PD method using reference source/PLL techniques. The basis of this method is to use a PLL in conjunction with a double balanced mixer (DBM) used for the PD.

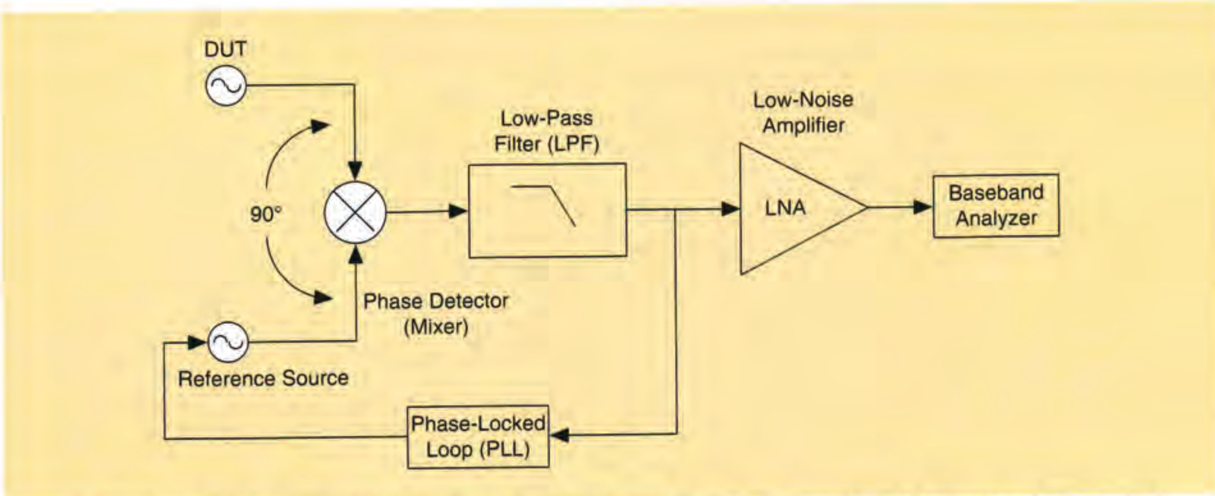

Figure 6. *The basic block diagram PD method using reference source/PLL techniques. Small fluctuations from nominal voltages are equivalent to phase variations. The phase lock loop keeps two signals in quadrature, which cancels carriers and converts phase noise to fluctuating dc voltage (courtesy of Agilent Technologies).*

TABLE 1. Correction factor if the phase noise of the reference oscillator is near the phase noise of DUT.

Δp/dB	0	2	4	6	8	10	15	20
$P_{\text{correction/dB}}$	3	2.12	1.46	0.97	0.64	0.4	0.14	0.04

The PLL compares the phases of two input signals and generates a third signal which is used to steer one of the input signals into phase quadrature with the other. When the phase of the input signals are aligned, the loop is said to be locked and the nominal output from the PD is zero. This voltage varies a little due to phase noise on the input signals. The noise present at the output of the mixer includes phase noise of both signals. If the noise from the reference oscillator is more than 20 dB lower than the noise from DUT, the main contributor for phase noise is the DUT.

As shown in Figure 6, two sources, one from the DUT and the other from the reference source, provide inputs to the mixer. Again the reference source is controlled such that it follows the DUT at the same carrier frequency (f_c) and in phase quadrature (90° out of phase) to null out the carrier power. The mixer sum frequency ($2f_c$) is filtered out by the low-pass filter (LPF), and the mixer difference frequency is 0 Hz (dc) with an average voltage output of 0 V when locked. The dc voltage fluctuations are directly proportional to the combined phase noise of the two sources. The noise signal is amplified using a low noise amplifier (LNA) and measured using a spectrum analyzer.

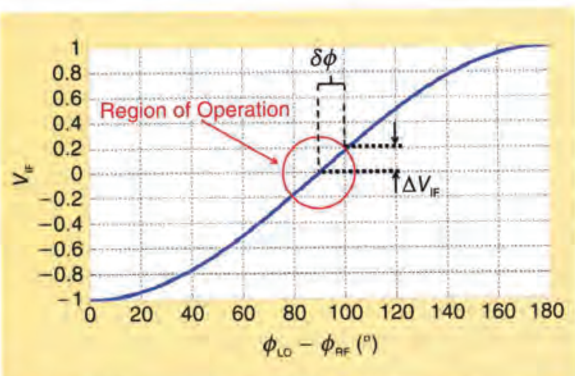

Figure 7. *The response of DBM as a PD varies as* $\cos(\Delta\emptyset + \pi)$, *(VIF) is reasonably linear in the region* $(\Delta\emptyset = \pi/2 + \delta\emptyset)$ *[31].*

The advantage of this method is broadband measurement capability for both fixed frequency and tunable oscillators. With only a few different double balanced mixers and suitable reference oscillators, noise on signals from 1 MHz to several tens of GHz can be measured. If the DUT is a tunable oscillator, the reference oscillator will then be a free running one and the DUT would be controlled with the PLL, and need a suitable PLL amplifier after the LPF. The limitation with this method is that it is not possible to know precisely which part of the noise comes from the reference and which from the DUT. Nevertheless, this problem is true for most measurement systems.

Usually, if the phase noise levels of the two signals are not that far from each other, a correction factor ($P_{\text{correction}}$) from 0 to 3 dB is subtracted from the measured result, where the highest number is used when the noise levels are equal [24]. The expression of the correction factor is given by [32]

$$P_{\text{correction}} = 10\log_{10}\left(1 + 10^{-\frac{\Delta P}{10}}\right), \tag{10}$$

where ΔP is the difference between the noises of the reference and the DUT in dB.

Table 1 shows the correction factors for different noise level differences.

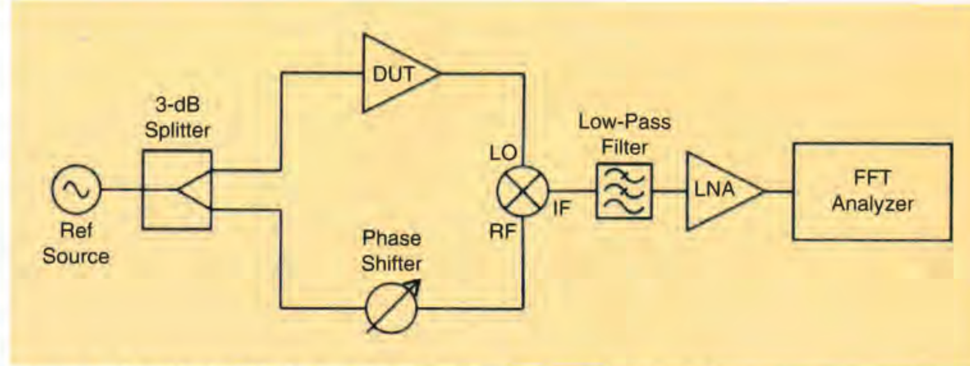

Figure 8. *A residual method set-up (simplified single channel residual phase noise measurement system).*

This method exhibits promising noise floor but the performance is dependent on DBM and reference source characteristics. The selection of a mixer as a PD is critical to the overall system performance. The noise floor sensitivity is related to the mixer input levels; therefore high power level mixers are preferred. But care must be taken to match mixer drive to available source power.

Choice of DBM as Phase Detector

Figure 7 exhibits typical DBM PD response curve, where V_{IF} varies as the cosine of the phase difference $\Delta\emptyset$ between LO and RF signals [31]. As shown in Figure 7, PD response (V_{IF}) is reasonably linear in the region $\Delta\emptyset$ where PD sensitivity ($\partial V_{IF}/\partial\emptyset$) is maximum, represented by

$$\Delta\emptyset = (\emptyset_{LO} - \emptyset_{RF}) = \left(\frac{\pi}{2} + \delta\emptyset\right). \tag{11}$$

The PD output $V_{IF}(t)$ is given by [5]

$$V_{IF}(t) = \pm V\cos[(\omega_R - \omega_L)t + \Delta\emptyset(t) + \pi]. \tag{12}$$

For the case where the mixer's two input signals are at the same frequency, $\omega_R = \omega_L$ and 90° out of phase, $V_{IF}(t)$ is

$$\Delta V_{IF}(t) = \pm V\sin\delta\emptyset(t), \tag{13}$$

where V is the peak amplitude (at $\Delta\emptyset = 0$ or π), $\Delta V_{IF}(t)$ is the instantaneous voltage fluctuations around dc, and $\delta\emptyset(t)$ is the instantaneous phase fluctuation.

For $\delta\emptyset(t) \ll 1$ rad, $\sin(\delta\emptyset(t)) \simeq \delta\emptyset(t)$, which describes a linear response region, the PD sensitivity varies linearly with maximum output voltage as

$$\Delta V_{IF}(t) = V\,\delta\emptyset = K_{\emptyset}\delta\emptyset, \tag{14}$$

where $K_{\emptyset}$ is the PD gain constant (volts/radian), equal to the slope of the mixer sine wave output at the zero crossing.

Choice of Reference Sources

The other critical component of the PD method is the reference source. As discussed in the "Direct Spectrum Technique" section, a spectrum analyzer measures the sum of noise from both sources. Therefore, the reference source must have lower phase noise than device under test, DUT. For practical purposes 10 dB margin is sufficient enough to ensure correct measurements within reasonable degree of accuracy. When a reference source with lower phase noise is unavailable then it is appropriate to use a source with comparable phase noise to the DUT. In this case, each source contributes equally to the total noise and 3 dB is subtracted from the measured value.

Advantages

- Excellent sensitivity for measuring low phase noise levels
- Wide signal frequency range
- Wide offset frequency range (0.01 Hz to 100 MHz)
- Rejects AM noise
- Frequency tracks slowly drifting sources.

Drawbacks

- Requires a very clean reference source that is electronically tunable,
- Measurement frequency bandwidth matched to the tuning range of the reference sources.
- Locking PLL bandwidth is very narrow, < 10% of the minimum offset frequency used in the measurement.
- Narrow PLL bandwidth cannot track a noisy source.
- Expensive and complex.

In conclusion, the PD method has excellent system sensitivity, but on the other hand its complexity (PLL and two oscillators are required) must be handled with care.

Residual Method

The methods shown thus far can be used to measure only oscillators. There are some methods for measuring two-port devices, and the residual method is one of them. It can be used for example to measure amplifiers, mixers, cables, and filters.

As shown in Figure 8, the output of a reference source is split with a power splitter. One branch is connected through the DUT to the mixer and the other branch through a phase shifter to the mixer. The phase shifter is adjusted until the phases are in quadrature, and the output of the mixer is measured with a spectrum analyzer.

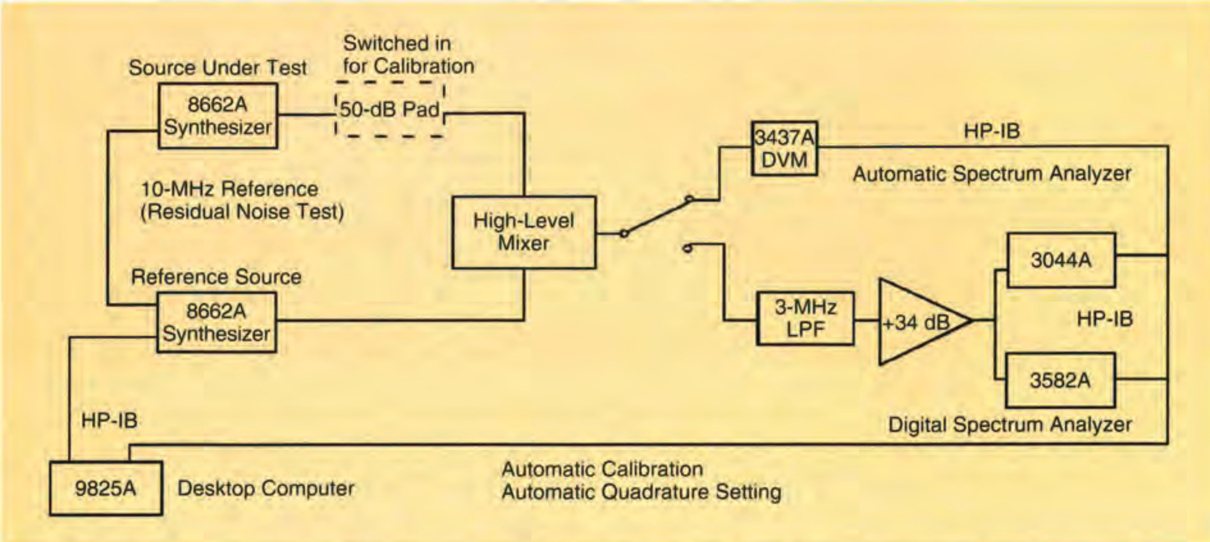

Figure 9. *An automatic system to measure residual phase noise of two 8662A synthesizers (courtesy of Hewlett-Packard Company).*

Because the noise from the reference source is coherent at the mixer input and the signals are in quadrature, it will be subject to some degree of cancellation.

The degree of cancellation improves as the signal path delay in the two arms of the bridge is minimized. The remaining phase noise at the mixer output is thus added by the DUT. When the DUT is relatively broadband (i.e. low delay) device having equal input and output frequencies, the need for a second device in the other bridge arm is eliminated. When the device is either narrowband, or is one with unequal input and output frequencies (a mixer frequency multiplier or divider etc.) identical devices must be used in both arms of the phase bridge.

The noise floor of a system utilizing this single channel measurement technique is highly dependent on and limited by the noise floors of the mixer, filters and low noise amplifier. This type of system can have a residual phase noise floor in the region of −180 dBc/Hz at high offset frequencies [2].

In residual measurement system phase, the noise of the common source might be insufficiently canceled due to improperly high delay-time differences between the two branches. It is therefore vitally important to match the delay times very closely.

A Residual Phase Noise Measurement System

Figure 9 shows a system that automatically measures the residual phase noise of the 8662A synthesizer [4]. It is a residual test, since both instruments use one common 10 MHz referenced oscillator. Quadrature setting is conveniently controlled by first offsetting the tuning of one synthesizer by a small amount, usually 0.1 Hz. The beat signal is then probed with a digital voltmeter and when the beat signal voltage is sufficiently close to zero, matching the synthesizer tuning commands to stop the phase slide between the synthesizers.

Two-Channel Cross-Correlation Technique

Figure 10 shows the diagram of the two-channel cross-correlation technique from Agilent [1], built around a similar measurement set-up as the PLL method except that there are three oscillators and the measurement involves performing cross-correlation operations among the outputs from each channel. It can be seen that there are two reference oscillators, one power splitter, two mixer/amplifier/PLL circuits and a cross-correlation FFT analyzer. The cross-correlation technique is used to minimize the noise contribution from mixer, filter and LNA from the measurement results.

This works because the noise from the DUT is common between both paths, but the noise contributed from each internal reference oscillator is independent. Thus over time, the noise contributions from the independent sources will show a zero cross-correlation. But the noise from the DUT will correlate, and ultimately dominate the output measurement (as desired).

The noise from the first reference feeds into the first phase noise detector and ends up on channel 1 of the cross-correlation FFT analyzer. The noise from the second reference passes through in the second phase noise detector and appears on channel 2 of the cross-correlation FFT analyzer. The output of the DUT is connected through a high isolation inductive power splitter to two mixer circuits where it is mixed with the signal from these two reference oscillators. The outputs of the mixer circuits are used for PLL circuits to lock the internal references in phase quadrature to the DUT input signal, as in the PLL method. The mixer

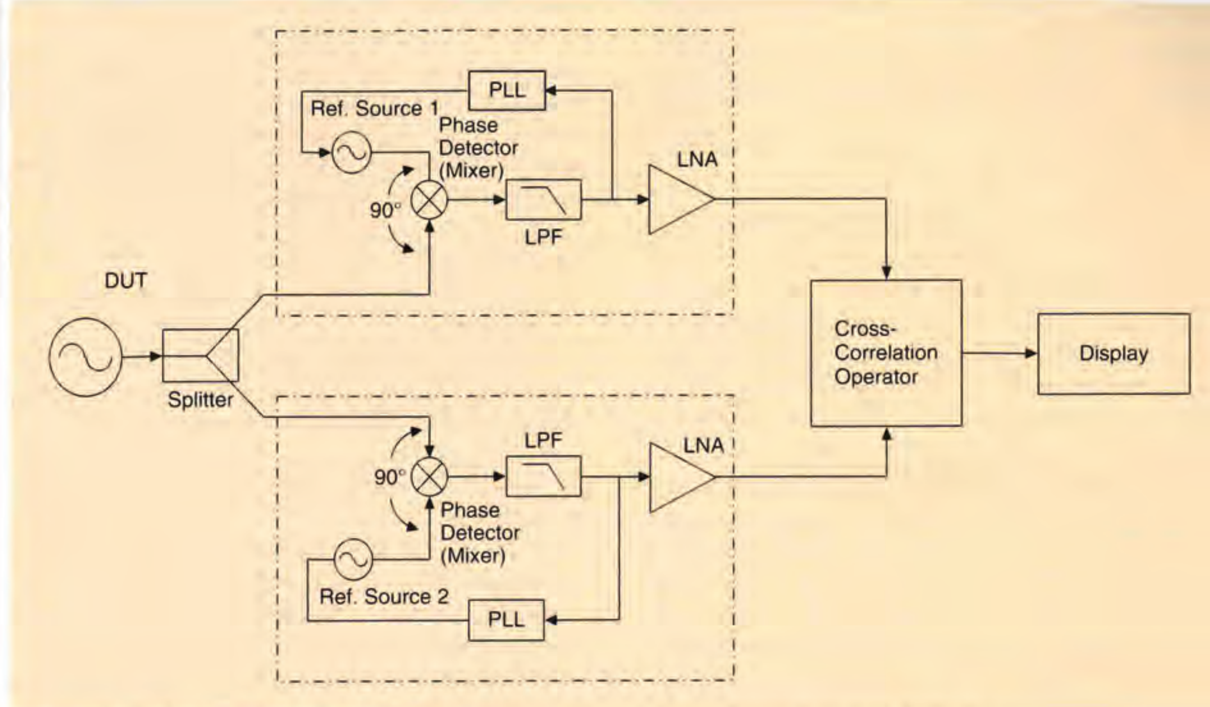

Figure 10. *The basic block diagram of a two-channel cross-correlation technique (courtesy: Agilent).*

output signals are then amplified, the dc is filtered away, and finally the signals are fed to two channels of the FFT analyzer to perform a cross-correlation measurement between the two output signals.

The noise from output of each mixer can be modeled using two noisy signals [36]–[39]

$$x(t) = a(t) + c(t) \overleftrightarrow{\mathrm{FFT}} X(f) = A(f) + C(f) \quad (15)$$

$$y(t) = b(t) + c(t) \overleftrightarrow{\mathrm{FFT}} Y(f) = B(f) + C(f), \quad (16)$$

where $a(t)$ and $b(t)$ are uncorrelated equipment noise present in each channel and $c(t)$ represents the correlated DUT noise. The cross-spectrum of these two signals after averaging over M samples is described by

$$\overline{S_{XY}} = \frac{1}{M}\sum_{m=1}^{m=M}[X_m \times Y_M^*], \quad (17)$$

where m represents the sample index and (*) implies the conjugate function.

From (15), (16) and (17) into (18) and (19),

$$\overline{S_{XY}} = \frac{1}{M}\sum_{m=1}^{m=M}[(A_m + C_m)\times(B_m + C_m)^*] \quad (18)$$

$$\overline{S_{XY}} = \frac{1}{M}\sum_{m=1}^{m=M}[(A_m B_m^*) + (A_m C_m^*) + (C_m B_m^*) + (C_m C_m^*)]. \quad (19)$$

Considering that there is no correlation between the noisy signals $a(t)$, $b(t)$ or $c(t)$, then as the number of averages increases the uncorrelated terms in the cross spectrum (AB, AC and CB) will tend toward zero. The only remaining term CC represents the power spectral density of the correlated DUT noise. When the analyzer is set to average, the common noise is kept, and the noise not common to both channels is attenuated and averaged away.

From (19) the DUT noise through each channel is coherent and is therefore not affected by the cross-correlation, whereas, the internal noises generated by each channel are incoherent and diminish through the cross-correlation operation at the rate of $\sqrt{M}$ (M = number of correlations)

$$[\mathrm{Noise}]_{\mathrm{meas}} = [\mathrm{Noise}]_{\mathrm{DUT}} + \left(\frac{[\mathrm{Noise}]_{\mathrm{chanel\#1}} + [\mathrm{Noise}]_{\mathrm{chanel\#2}}}{\sqrt{M}}\right), \quad (20)$$

where $[\mathrm{Noise}]_{\mathrm{meas}}$ is the total measured noise at the display; $[\mathrm{Noise}]_{\mathrm{DUT}}$ the DUT noise; $[\mathrm{Noise}]_{\mathrm{chanel\#1}}$ and $[\mathrm{Noise}]_{\mathrm{chanel\#2}}$ are the internal noise from channels 1 and 2, respectively; and M the number of correlations.

From (20), the two-channel cross-correlation technique achieves superior phase noise measurement capability but the measurement speed suffers when increasing the number of correlations. This method offers 15–20 dB improved phase noise measurement sensitivity when compared to the reference source/PLL method described above, so it can be used to measure oscillators with ultra-low phase noise. It is even possible to measure oscillators with better noise performance than the reference oscillators because phase noises from the reference oscillators are suppressed considerably.

The improved dynamic range and noise floor of the cross-correlation phase noise measurement technique comes at price. Usually many samples are needed in order to average out the uncorrelated noise. The measuring yardstick of the confidence interval of a phase noise detector is expressed by [26]:

$$S_{\phi}^{s}(f) = S_{\phi}^{m}(f)\left(1 \pm \frac{1}{\sqrt{n}}\right)$$
$$= \mp 10\% \,(\text{for } n = 100) \rightarrow \text{For single-channel} \quad (21)$$

$$S_{\phi}^{x}(f) = S_{\phi}^{m}(f)\left(1 \pm \frac{2S_{\phi}^{s}}{\sqrt{n}}\right)$$
$$= \mp 10\% \,(\text{for } n = 20{,}000) \rightarrow \text{For dual-channel}, \quad (22)$$

where x = cross–correlation, m = measured (noise), s = single channel, and n = number of samples.

Equation (21) shows that for a single channel the confidence interval is ±10% for 100 samples. Equation (22) shows that to obtain the same confidence interval for a phase noise measurement 10 dB below the single channel noise floor 20,000 samples are required. Indeed, the dual channel or cross-correlation method of phase noise results in a lower floor than the standard single channel method but there is a cost of measurement speed. From (21) and (22), more averages are required to achieve the same level of confidence in a measurement for dual-channel cross-correlation method. The advantage of lower noise floor using the cross-correlation method provides a level of characterization of extremely low noise crystal oscillators, which was not possible using the single channel method. The practical value of the noise floor is given by

$$[L(f)_{SSB}] = -177 + N_a - P_i, \quad (23)$$

where N_a is the noise figure and P_i is the power available.

Today, the cross-correlation process is the only technique that allows close to thermal noise floor measurements below −177 dBc/Hz at far offset from the carrier, and with 20 dB of DUT output power can provide a noise floor better than −195 dBc/Hz provided the DUT output buffer stage is low noise amplifier and can handle the 20 dBm power. However, this improvement of 20 dB is based on 100,000 correlations which results in a long measurement time.

Advantages

- Best sensitivity for measuring low phase noise levels.
- Wide signal frequency range.
- Wide offset frequency range (0.01 Hz to 100 MHz).
- Frequency tracks slowly drifting sources.
- Rejects AM noise.

Disadvantages

- Complexity: Requires two very clean reference sources that are electronically tunable.
- Long measurement times when very low noise is being measured.
- Measurement frequency bandwidth matched to the tuning range of the reference sources.

Conventional Phase Noise Measurement System (Hewlett-Packard)

This section is based on published Hewlett-Packard material [1], described here to give brief insights about the working principle of the early, very low phase noise measurement equipment (during the 1980s) and subsequently the development of modern automated test systems [4]. Here we present more mathematical details of this measurement technique.

The most sensitive method to measure the spectral density of phase noise $S_{\Delta\theta}(f_m)$ requires two sources—one or both of them may be the device(s) under test—and a double balanced mixer used as a PD. The RF and LO input to the mixer must be in phase quadrature, indicated by 0 Vdc at the mixer IF port. Good phase quadrature assures maximum phase sensitivity K_θ and minimum AM sensitivity. With a linearly operating mixer, K_θ equals the peak voltage of the sinusoidal beat signal produced when both sources are frequency offset (Figure 11). When both signals are set in quadrature, the voltage ΔV at the IF port is proportional to the fluctuating phase difference between the two signals.

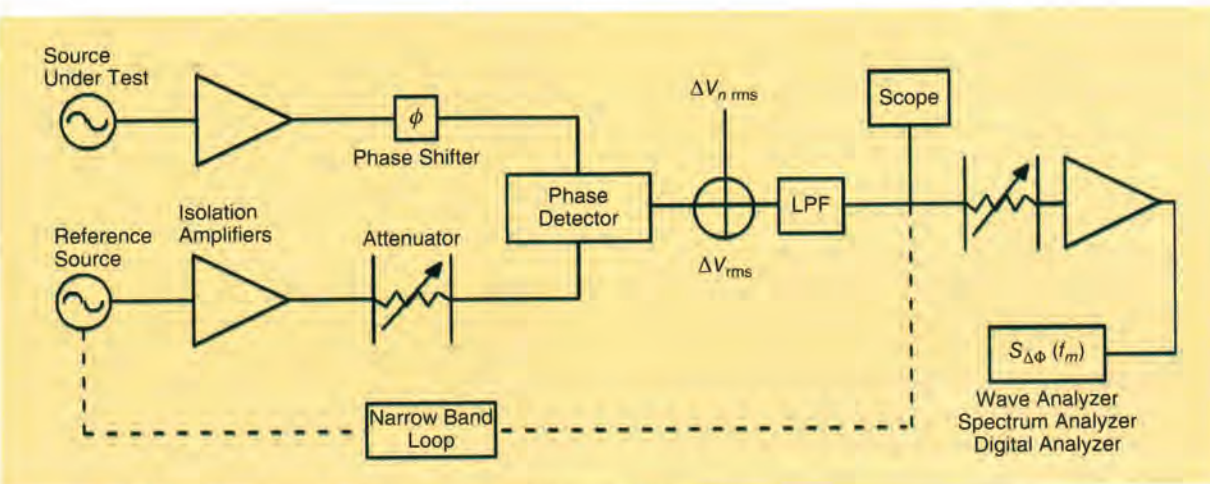

Figure 11. *A phase noise system with two sources maintaining phase quadrature.*

$$\Delta\theta_{\mathrm{rms}} = \frac{1}{K_\theta V_{\mathrm{rms}}}, \quad (24)$$

$$S_{\Delta\theta}(f_m) = \frac{(\Delta V_{\mathrm{rms}})^2 (1\,\mathrm{Hz})}{V_{B\,\mathrm{peak}}^2} \frac{1}{2} \frac{(\Delta V_{\mathrm{rms}})^2 (1\,\mathrm{Hz})}{V_{B\,\mathrm{rms}}^2}, \quad (25)$$

$$\mathcal{L}(f_m) = \frac{1}{2} S_{\Delta\theta}(f_m) = \frac{1}{4} \frac{(\Delta V_{\mathrm{rms}})^2 (1\,\mathrm{Hz})}{V_{B\,\mathrm{rms}}^2}, \quad (26)$$

where K_θ is PD constant and $V_{B\,\mathrm{peak}}$ for sinusoidal beat signal

Calibrations required of the wave analyzer or spectrum analyzer can be read from the equations above. For a plot of $\pounds(f_m)$ the 0-dB reference level is to be set 6 dB above the level of the beat signal. The −6-dB offset has to be corrected by +1.0 dB for a wave analyzer and by +2.5 dB for a spectrum analyzer with log amplifier followed by an averaging detector. In addition, noise bandwidth corrections likely have to be applied to normalize to 1 Hz bandwidth.

Since the phase noise of both sources is summed together in this system, the phase noise performance of one of them needs to be known for definite data on the other source. Frequently, it is sufficient to know that the actual phase noise of the dominant source cannot deviate from the measured data by more than 3 dB. If three unknown sources are available, three measurements with three different source combinations yield sufficient data to calculate accurately each individual performance.

Figure 11 indicates a narrowband phase-locked loop that maintains phase quadrature for sources that are not sufficiently phase stable over the period of the measurement. The two isolation amplifiers are to prevent injection locking of the sources to each other. The noise floor of the system is established by the equivalent noise voltage ΔV_n at the mixer output. It represents mixer noise as well as the equivalent noise voltage of the following amplifier:

$$\mathcal{L}_{\mathrm{system}}(f_m) = \frac{1}{4} \frac{(\Delta V_{n\,\mathrm{rms}})^2 (1\,\mathrm{Hz})}{V_{B\,\mathrm{rms}}^2}. \quad (27)$$

Wideband noise floors close to −180 dBc can be achieved with a high-level mixer and a low-noise port amplifier. The noise floor increases with f_m^{-1} due to the flicker characteristic of ΔV_n. System noise floors of −166 dBc/Hz at 1 kHz have been realized.

To get this excellent performance, the PD/PLL method is complex and requires significant calibration. In measuring low-phase-noise sources, a number of potential problems have to be understood to avoid erroneous data. These include:

In a production environment, the objective is fast throughput for product phase noise performance testing.

- When two sources are phase locked to maintain phase quadrature, it has to be ensured that the lock bandwidth is significantly lower than the lowest Fourier frequency f_m of interest, unless the test set takes into account (as many do) the loop suppression response.
- Even with no apparent phase feedback, two sources can be phase locked through injection locking, resulting in suppressed close-in phase noise and causing a measurement error. This can normally be avoided with the use of high isolation buffer amplifiers or frequency multipliers.
- AM noise of the RF signal can come through if the quadrature setting is not maintained accurately.
- Deviation from the quadrature setting also lowers the effective PD constant.
- Nonlinear operation of the mixer results in a calibration error.
- Need for low harmonic content: A nonsinusoidal RF signal causes K_θ to deviate from $V_{B\,\mathrm{peak}}$.
- The amplifier or spectrum analyzer input can be saturated during calibration or by high spurious signals such as line frequency multiples.
- Closely spaced spurious signals such as multiples of 60 Hz may give the appearance of continuous phase noise when insufficient resolution bandwidth and averaging are used on the spectrum analyzer.
- Impedance interfaces must remain unchanged when transitioning from calibration to measurement.

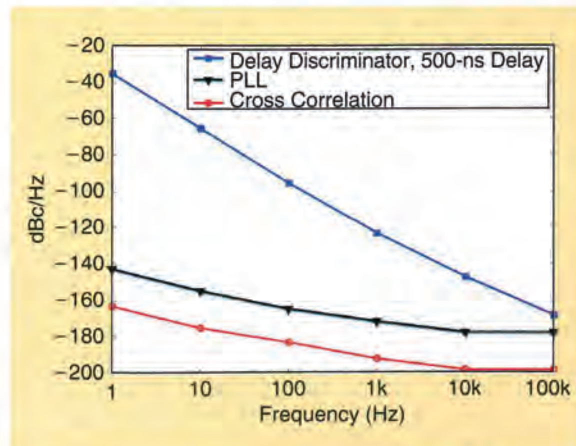

Figure 12. *Phase noise plots and noise floor for three techniques (PLL, delay line, and cross-correlation) [31].*

Noise in Circuits and Semiconductors [9], [10], and [22]

In general, phase noise describes how the frequency of an oscillator varies in a short time scale. The long term frequency stability is called drift, and it must also be considered during the measurement process. The output frequency of an oscillator takes finite time to stabilize after the oscillator has been started, and this drift can be significant. The output frequency also usually drifts noticeably during measurements, especially in the case of free running oscillators. This drift is a real problem because during the measurements the system must be able to lock to the carrier or carrier must be stable enough; therefore without the carrier tracking mechanism, measurement is a nightmare. Microwave applications generally use bipolar transistors, and following are their noise contributions.

Johnson Noise

- The Johnson noise (thermal noise) is due to the movement of molecules in solid devices called Brown's molecular movements.
- This noise voltage is expressed as $v_n^2 = 4kT_0 RB(\text{emf})(\text{volt}^2)$.
- The power of thermal noise can thus be written as

$$\text{Noise Power} = \frac{v_n^2}{4R} = kT_0B(W).$$

- It is most common to do noise evaluations using a noise power density, in watts per hertz. We get this by setting $B = 1$ Hz. Then we get

$$\text{For } B = 1\,\text{Hz}, \text{Noise Power} = kT_0$$

$$T = 290\,K \text{ and } k - \text{Boltzmann's constant} = 1.38 \times 10^{-23} J/K$$

$$\text{by Thevinin, Noise Power} = 1.38 \times 10^{-23} \times 290 = 4 \times 10^{-23} W.$$

- Noise floor below the carrier for zero dBm output is given by

$$L(\omega) = 10\log\left(\frac{v_n^2/R}{1mW}\right) = -173.97\,\text{dBm or about } -174\,\text{dBm}.$$

- In order to reduce this noise, the only option is to lower the temperature since noise power is directly proportional to temperature.
- The Johnson noise sets the theoretical noise floor.

Planck's Radiation Noise

- The available noise power does not depend on the value of resistor but it is a function of temperature T. The noise temperature can thus be used as a quantity to describe the noise behavior of a general lossy one-port network.
- For high frequencies and/or low temperature, a quantum mechanical correction factor has to be incorporated for the validation of equation. This correction term results from Planck's radiation law, which applies to blackbody radiation.

$$P_{av} = kT.\Delta f$$

$$P_{av} = kT.\Delta f \cdot p(f,T),$$

with

$$p(f,T) = \left[\frac{hf}{kT} \Big/ \left(e^{\left(\frac{hf}{kT}\right)} - 1\right)\right],$$

where $h = 6.626.10^{-34} J - s$, Planck's constant.

Schottky/Shot Noise

The Schottky noise occurs in conducting PN junctions (semiconductor devices) where electrons are freely moving. The root mean square (RMS) noise current is given by

$$\overline{i_n^2} = 2 \times q \times I_{dc}$$

$$P = i_n^2 \times Z,$$

where q is the charge of the electron, P is the noise power, and I_{dc} is the dc bias current; and Z is the termination load (can be complex).

Since this noise process is totally different from other noise processes, this noise is independent from all others.

Flicker Noise

- The electrical properties of surfaces or boundary layers are influenced energetically by states, which are subject to statistical fluctuations and therefore, lead to the flicker noise or $1/f$ noise for the current flow.
- $1/f$-noise is observable at low frequencies and generally decreases with increasing frequency f according to the $1/f$-law until it will be covered by frequency independent mechanism, like thermal noise or shot noise.
- **Example**: The noise for a conducting diode is bias dependent and is expressed in terms of AF and KF

$$\langle i_{Dn}^2 \rangle_{AC} = 2qI_{dc}B + KF\frac{I_{DC}^{AF}}{f}B.$$

- The AF term is a dimensionless quantity and a bias dependent curve fitting parameter. This term has a value generally within the range of one to three and a typical value of two.
- The KF value ranges from $1E^{-12}$ to $1E^{-6}$ and defines the flicker corner frequency.

Transit Time and Recombination Noise

- When the transit time of the carriers crossing the potential barrier is comparable to the periodic signal, some carriers diffuse back and this causes noise. This is really seen in the collector area of NPN transistor at high frequencies.
- The electron and hole movements are responsible for this noise. The physics for this noise has not been fully established.

Avalanche Noise

- This process begins when the applied reverse bias approaches the breakdown voltage and does not happen for any reverse bias.
- When the high reverse bias is applied to semiconductor junction, the normally small depletion region expands rapidly.
- The free holes and electrons then collide with the atoms in depletion region, thus ionizing them and produce spiked current called the avalanche current. The spectral density of avalanche noise is mostly flat. At higher frequencies the junction capacitor with lead inductance acts as a low-pass filter.
- Zener diodes are based on Zener effect used as voltage reference sources. The avalanche noise associated with Zener diodes needs to be reduced by big bypass capacitors!

- Noise from power supplies for devices under test can be a dominant contributor of error in the measured phase noise.
- Peripheral instrumentation such as an oscilloscope, analyzer, counter, or DVM can inject noise.
- Microphonic effects may excite significant phase noise in devices.

Despite all these hazards, automatic test systems now exist and operate successfully [8]. Oscillator manufacturers and users who frequently need to evaluate the performance of ultra low phase noise oscillators, at some point, recognize that their phase noise test systems could be primarily improved in the following aspects:

Figure 13. *Ultra-low phase noise setup in Faraday cage: phase noise measurement of 100-MHz OCXO using Agilent E5052B, Rohde & Schwarz FSUP 26, and Holzworth for understanding the speed and dynamic ranges of the equipment.*

1) accuracy
2) speed of test
3) large dynamic range and lower noise floor
4) reliability and repeatability of test data
5) range, ease of use and data retrieval
6) cost (though high performance test systems will never be cheap).

General Discussion

Characterizing the phase noise of a system or component is not necessarily very easy. Many different approaches are possible, but the key is to find the best approach for the measurement requirements at hand. Practically, it is advisable to use the cross-correlation approach for the best sources so that keeping them locked is easy during measurement cycle. In principle, each reference is locked to track the DUT, therefore PLL bandwidth needs to be monitored for reliable and accurate measurement. Usually, corrections for PLL bandwidth works to some degree, but corrections beyond certain limit have more errors, leading to inaccurate phase noise measurement of the DUT. One of the weaknesses, with the cross-correlation method is that, many measurements must be made and the average calculated between them. Thus the measurement takes longer, and the DUT must be kept locked for a longer time. Usually, one sweep takes approximately 10 s, and the required amount of sweep is 2^m where $m > 2$ but for a noisy source this may not be easy. Hence, this method is most suitable for measuring low noise oscillators having a small frequency drift.

A survey of some of the more common topologies along with some possible trouble spots helps one to review and keep in mind the advantages and limitations of each approach. Figure 12 Shows phase noise plots and noise floor for three-phase noise measurement techniques (delay line, PLL and cross correlation).

Conclusion

There are many areas in which design engineers can be tricked into false readings or frustrated with the process of trying to achieve a good phase noise measurement. Characterizing the phase noise of a system or component is generally very difficult. Many different approaches are possible, but the key is to find the best approach for the measurement requirements at hand.

It is especially pertinent in a production environment, where measurement time and accuracy of each measurement becomes critical. Several test methods and test instruments are investigated as shown in Figure 13. There's no "one size fits all" solution, for e.g., measuring a phase noise value of –130 dBc/Hz at 1 Hz offset from the carrier, and/or achieving a measurement noise floor better than –190 dBc/Hz is challenge, using existing test equipment and methods.

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